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Solitons in Optical Bistability

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Solitons in optical bistability

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Concentric spatial rings (saturable media) or symmetrically disposed filament-like structures (Kerr media) are predicted to slowly evolve across the turned-on cylindrical spot of the two-dimensional beam profile in a bistable optical resonator.

INTRODUCTION

In this paper we report on a novel nonlinear dynamical phenomenon involving the switching of an optical beam between low and high transmission states of a bistable optical ring resonator. While the non-linear intensity-dependent phase change encoded across the beam profile may be small in a single pass, a significant cumulative phase change may occur over many resonator passes due, in particular, to a large build-up of the intracavity field. If diffractive coupling is weak, only the more intense central portion of the beam exceeds threshold for switching to the high transmission branch. The end result is an intense 'on' spot with a sharp gradient at its outer edge. Furthermore, if the driving laser frequency is tuned to the self-focusing side of the atomic transition, strong diffraction will cause spatial rings to appear on the outer edge of the 'on' spot and slowly evolve towards the centre of the beam (Moloney & Gibbs 1982). The transverse spatial rings may reach an asymptotically stable state or undergo slow periodic oscillations. At higher input driving-field amplitudes the rings may undergo a bifurcation to periodic and chaotic temporal motion while retaining spatial coherence. The evolution of spatial rings is fundamentally different for saturable and Kerr media, and if a single transverse spatial dimension is assumed, the rings can be identified with transverse solitary waves and solitons, respectively (McLaughlin *et al.* 1983). Recently, we have extended our studies to the full two-dimensional transverse-beam profile and preliminary numerical results will be reported here. Unlike conventional self-focusing experiments, the shapes (heights and widths) of the transverse spatial rings are not dependent on initial conditions, but are determined as fixed points of an infinite-dimensional map.

THEORY

In the good-cavity limit, the atomic medium (two-level atom) variables may be adiabatically eliminated from the Maxwell–Bloch equations to lead to the following nonlinear evolution equation for propagation of the electromagnetic field through the medium:

$$2\mathrm{i}\frac{\partial G_n}{\partial \zeta}+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)G_n-\frac{G_n}{1+2|G_n|^2}=0, \quad (1)$$

and the ring resonator boundary conditions become

$$G_n(x,y,0)=a(x,y)+R\mathrm{e}^{\mathrm{i}kL}G_{n-1}(x,y,p), \quad G_0=0. \quad (2)$$

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These equations together constitute an infinite-dimensional map in the discrete time variable n, where n counts the number of circuits of the field around the resonator. G_n is the normalized intracavity field amplitude; (x, y) and ζ refer to suitably normalized coordinates in the transverse and propagation directions, respectively. These equations are written in non-dimensional form and are discussed by Moloney & Gibbs (1982) and by McLaughlin *et al.* (1983).

Equations (1) and (2) are solved as follows. The initial input beam profile $a(x, y)$, which we assume to be gaussian, acts as initial data for the nonlinear evolution equation (1). This equation is solved over the effective nonlinear medium length p and the result substituted into (2) determines the new initial data for (1). This procedure is repeated until the system reaches an asymptotic state, which may be stable or unstable.

The fact that solitons may arise as asymptotic states of (1) is most easily seen if we assume that the laser is tuned far from any medium resonance (Kerr limit, $|G_n| \ll 1$) and if we drop one transverse dimension. In this limit (1) simplifies to

$$2\mathrm{i}\frac{\partial G_n}{\partial \zeta}+\frac{\partial^2 G_n}{\partial y^2}-(1-2|G_n|^2)\,G_n = 0, \tag{3}$$

which is the well-known nonlinear Schrödinger equation with soliton solutions

$$G_n(y, \zeta) = \lambda \operatorname{sech}(\lambda y) \exp\{\tfrac{1}{2}\mathrm{i}\,(\lambda^2-1)\,\zeta+\mathrm{i}\gamma\}.$$

Here λ specifies the amplitude and width $(1/\lambda)$ of the soliton and γ is its phase. The analogue of (3) for a saturable medium is non-integrable and admits solitary wave solutions.

The three-dimensional problem

We now present some preliminary results of a numerical study of (1) and (2) for a two-dimensional transverse gaussian input profile $a(x, y)$. Figure 1 shows the dynamical evolution of one quadrant of the two-dimensional beam profile $|G_n(x, y, p)|$ at every 20th circuit of the ring resonator (i.e. $n = 20$, 40, etc.). On the 20th circuit, the sharp gradient is evident at the outer edge of the cylindrical 'on' spot. By the 40th circuit, the transverse solitary waves are already well developed as outer concentric rings and are slowly evolving towards the centre of the beam. The two outer rings appear to quickly stabilize while the centre keeps oscillating. In fact, at the parameter values specified in this figure the asymptotic state appears to be a slow recurrent periodic oscillation. We predict for this saturable case that an amplitude threshold exists below which the rings are unstable and break into filaments while they remain as concentric rings at large amplitudes. Physically, this is reasonable, as large-amplitude rings saturate the nonlinearity and the system is quasi-linear, while at low amplitude the saturable nonlinearity is 'Kerr-like' and filamentation is expected to occur. Unlike the Kerr case, however, the filaments cannot critically focus because of saturation. Details of the analysis and further results will be presented by McLaughlin *et al.* (1984).

References

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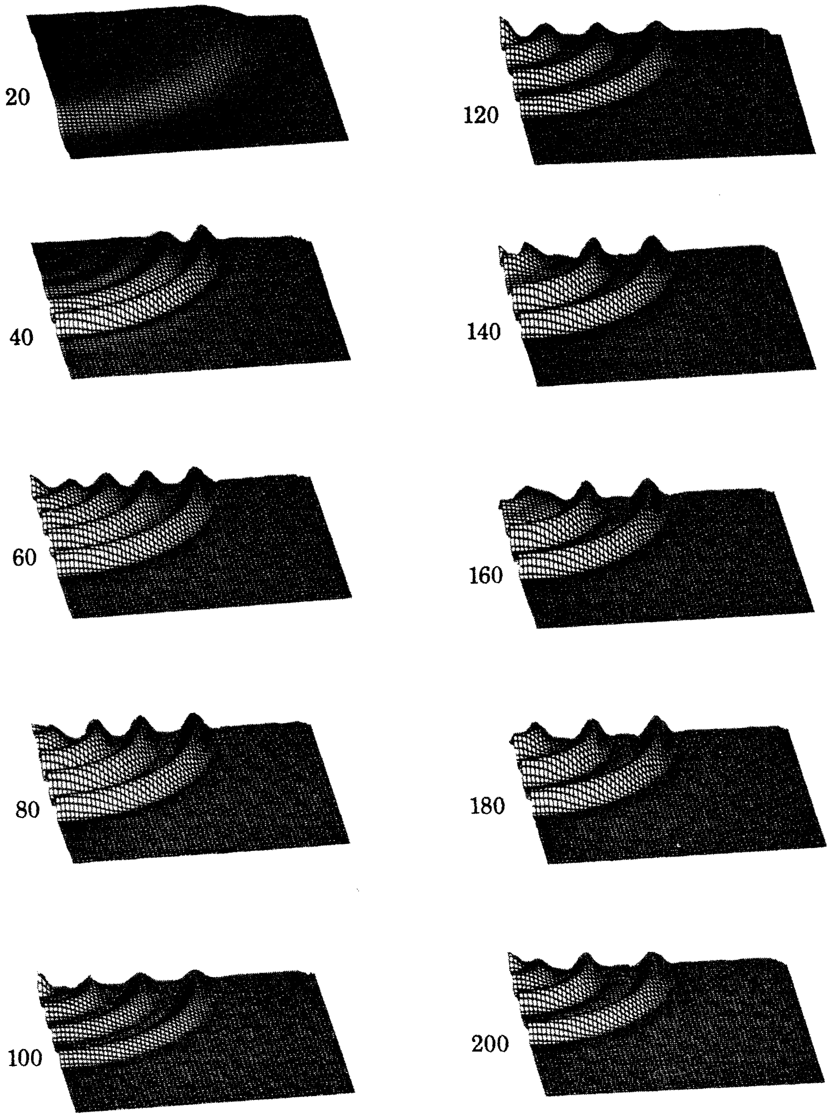

FIGURE 1. Time evolution of one quadrant of the two-dimensional intensity profile as the beam switches from a low to a high transmission state of the bistable system. The numbers indicate the resonator pass. The nonlinearity is saturable as in (1) with kL = 0.2, F = 100 and input peak intensity $|a(0,0)|^2 = 0.0375$. These parameters are explicitly defined by McLaughlin *et al.* (1983).

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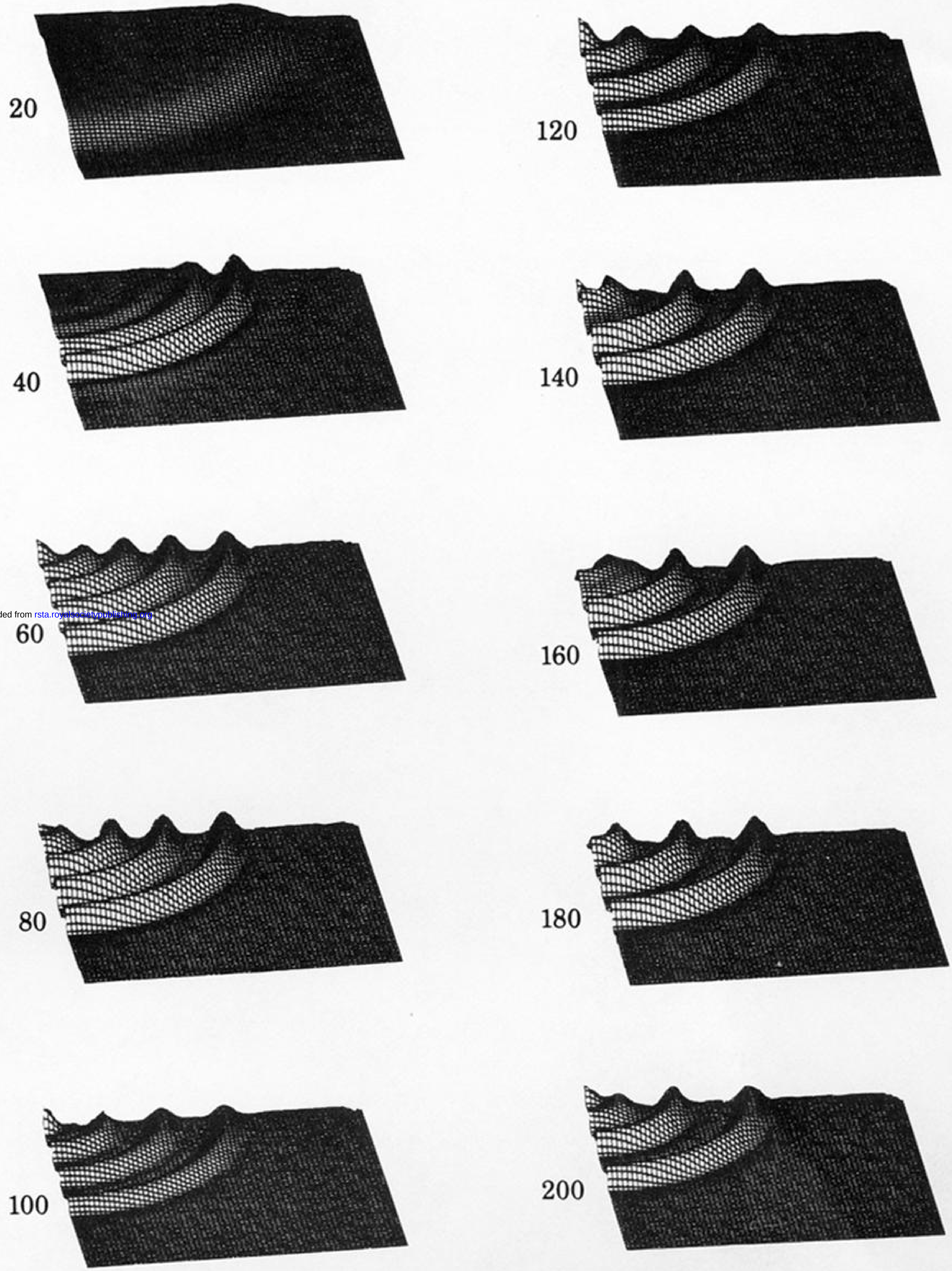

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